Joule's Law

Recall that the **work** done **on** charge Q **by** an electric field in moving the charge along some **contour** C is:

$$W = Q \int_C \mathbf{E}(\bar{\mathbf{r}}) \cdot \overline{d\ell}$$

Q: *Say instead of one charge Q, we have a steady* ***stream*** *of charges (i.e., electric current) flowing along contour C?*

A: We would need to determine the **rate** of work **per unit time**, i.e., the **power** applied by the field to the current.

Recall also that the **time derivative** of work is power!

$$P = \frac{dW}{dt} = \frac{d}{dt}\left(Q \int_C \mathbf{E}(\bar{\mathbf{r}}) \cdot \overline{d\ell} \right)$$

Since the electric field is **static**, we can write:

$$\begin{aligned} P &= \frac{dQ}{dt} \int_C \mathbf{E}(\bar{\mathbf{r}}) \cdot \overline{d\ell} \\ &= I \int_C \mathbf{E}(\bar{\mathbf{r}}) \cdot \overline{d\ell} \end{aligned}$$

But look! The **contour integral** we know is equal to the **potential difference** V between either end of the contour. Therefore:

$$P = I \int_C \mathbf{E}(\bar{r}) \cdot \overline{d\ell}$$
$$= I\,V$$

Look familiar!?

The **power** delivered to charges by the field is equal to the **current** I flowing along the contour, **times** the **potential difference** (i.e., voltage V) across the contour.

Consider now the power delivered in some **volume** V, say the volume of a resistor. Recall the electric field has units of **volts/m**, and the current density has units of **amps/m^2**.

We find therefore that the **dot product** of the electric field and the current density is a **scalar** value with units of Watts/m^3. We call this scalar value the **power density**:

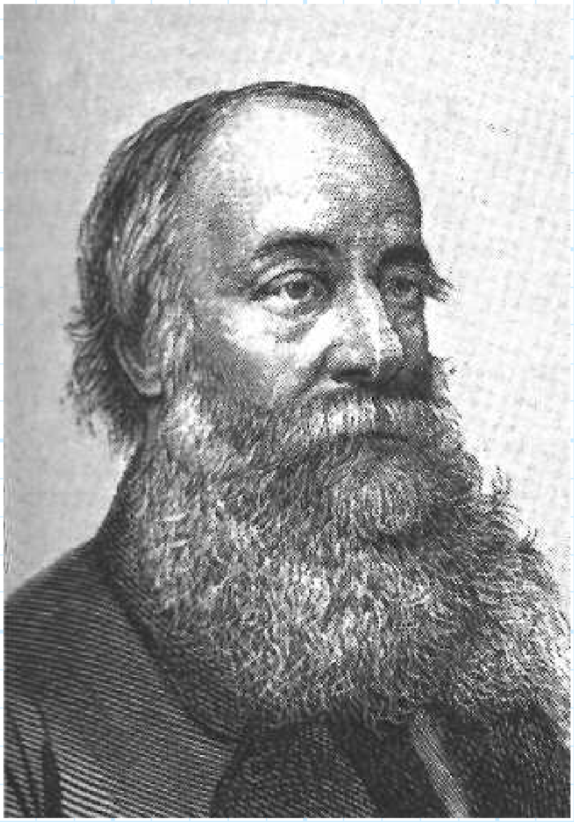
$$\text{power density} = \mathbf{E}(\bar{r}) \cdot \mathbf{J}(\bar{r}) \qquad \left[\left(\frac{\text{V}}{\text{m}}\right)\left(\frac{\text{A}}{\text{m}^2}\right) = \frac{\text{W}}{\text{m}^3}\right]$$

Integrating power density over some volume V gives the **total power** delivered by the field **within that volume**:

$$P = \iiint_V \mathbf{E}(\bar{\mathbf{r}}) \cdot \mathbf{J}(\bar{\mathbf{r}})\, dv$$

$$= \iiint_V \sigma(\bar{r}) \left|\mathbf{E}(\bar{\mathbf{r}})\right|^2 dv \qquad [\mathrm{W}]$$

$$= \iiint_V \frac{1}{\sigma(\bar{r})} \left|\mathbf{J}(\bar{\mathbf{r}})\right|^2 dv$$



James Prescott Joule (1818-1889), born into a well-to-do family prominent in the brewery industry, studied at Manchester under Dalton. At age twenty-one he published the "I-squared-R" law which bears his name. Two years later, he published the first determination of the mechanical equivalent of heat. He became a collaborator with Thomson and they discovered that the temperature of an expanding gas falls. The "Joule-Thomson effect" was the basis for the large refrigeration plants constructed in the 19th century (but not used by the British brewery industry). Joule was a patient, methodical and devoted scientist; it became known that he had taken a thermometer with him on his honeymoon and spent time attempting to measure water temperature differences at the tops and bottoms of waterfalls.

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